# SAMPLE QUESTIONS FOR THE SECOND MIDTERM EXAMINATION PART 2

1) Consider the system of linear equations

$$\begin{array}{rcrcrcrcrcr} & + & y & - & z & + & t & + & 5w & = & 5 \\ 3x & + & 3y & + & 3z & + & t & & & = & 6 \\ x & + & 2y & & & + & t & + & 4w & = & 6 \\ 2x & - & y & + & 5z & & & - & 8w & = & -4 \end{array}$$

(a) Write the augmented matrix of the system.

(b) Find the reduced row echelon form of the augmented matrix you found in part (a).

(c) Find the set of solutions of the system.

2) Consider the system of linear equations

$$\begin{array}{rcrcrcrcr} 2x & - & y & & & + & 5t & = & 4 \\ x & - & y & + & z & + & 6t & = & 0 \\ 3x & - & 2y & + & 2z & + & 14t & = & 3 \end{array}$$

(a) Write the augmented matrix of the system.

(b) Find the reduced row echelon form of the augmented matrix you found in part (a).

(c) Find the set of solutions of the system.

3) Consider the system of linear equations

$$\begin{array}{rcrcrcrcrcrcr} 2x & - & 2y & - & z & + & s & + & 2t & + & w & = & -6 \\ -3x & + & 3y & - & z & & & - & 7t & + & 2w & = & -3 \\ x & - & y & + & z & & & + & 3t & - & w & = & 3 \\ x & & & + & z & & & + & t & - & w & = & 4 \end{array}$$

(a) Write the augmented matrix of the system.

(b) Find the reduced row echelon form of the augmented matrix you found in part (a).

(c) Find the set of solutions of the system.

4) Consider the system of linear equations

$$\begin{array}{rcrcrcrcr} x & + & y & + & 2z & + & 3t & = & 13 \\ x & - & 2y & + & z & + & t & = & 8 \\ 3x & + & y & + & z & - & t & = & 1 \end{array}$$

(a) Write the augmented matrix of the system.

(b) Find the reduced row echelon form of the augmented matrix you found in part (a).

(c) Find the set of solutions of the system.

5) Let $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix}$

(a) Find cofactor of each entry of $A$.

(b) Find $\det(A)$ by using cofactor expansion with respect to second row. Determine whether $A$ is invertible.

(c) Find $\det(A)$ by converting $A$ to an upper triangular matrix.

(d) Find the adjoint of $A$ ($Adj(A)$).

(e) Find $A^{-1}$ if $A$ is invertible, by using $Adj(A)$.

(f) Find $A^{-1}$ if $A$ is invertible, by using elementary row operations.

(g) Find the solution of $Ax = b$ by using $A^{-1}$ if $A$ is invertible.

(h) Find the solution of $Ax = b$ by using Cramer's Rule if it is possible.

6) Let $A = \begin{bmatrix} -1 & 4 & 2 \\ 2 & -1 & 5 \\ 0 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 11 \\ -11 \\ 11 \end{bmatrix}$

(a) Find cofactor of each entry of $A$.

(b) Find $\det(A)$ by using cofactor expansion with respect to second row. Determine whether $A$ is invertible.

(c) Find $\det(A)$ by converting $A$ to an upper triangular matrix.

(d) Find the adjoint of $A$ ($Adj(A)$).

(e) Find $A^{-1}$ if $A$ is invertible, by using $Adj(A)$.

(f) Find $A^{-1}$ if $A$ is invertible, by using elementary row operations.

(g) Find the solution of $Ax = b$ by using $A^{-1}$ if $A$ is invertible.

(h) Find the solution of $Ax = b$ by using Cramer's Rule if it is possible.

7) Let Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 7 & 8 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 3 \\ -15 \end{bmatrix}$

(a) Find cofactor of each entry of $A$.

(b) Find $\det(A)$ by using cofactor expansion with respect to second row. Determine whether $A$ is invertible.

(c) Find $\det(A)$ by converting $A$ to an upper triangular matrix.

(d) Find the adjoint of $A$ ($Adj(A)$).

(e) Find $A^{-1}$ if $A$ is invertible, by using $Adj(A)$.

(f) Find $A^{-1}$ if $A$ is invertible, by using elementary row operations.

(g) Find the solution of $Ax = b$ by using $A^{-1}$ if $A$ is invertible.

(h) Find the solution of $Ax = b$ by using Cramer's Rule if it is possible.

8) Let $A = \begin{bmatrix} 2 & -1 & -7 \\ -2 & 0 & 4 \\ 3 & 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

(a) Find cofactor of each entry of $A$.

(b) Find $\det(A)$ by using cofactor expansion with respect to second row. Determine whether $A$ is invertible.

(c) Find $\det(A)$ by converting $A$ to an upper triangular matrix.
(d) Find the adjoint of $A$ ($Adj(A)$).
(e) Find $A^{-1}$ if $A$ is invertible, by using $Adj(A)$.
(f) Find $A^{-1}$ if $A$ is invertible, by using elementary row operations.
(g) Find the solution of $Ax = b$ by using $A^{-1}$ if $A$ is invertible.
(h) Find the solution of $Ax = b$ by using Cramer's Rule if it is possible.

9) Let $A$ be a $5 \times 5$, $B$ be a $3 \times 3$ and $C$ be a $4 \times 4$ matrices with $|A| = 2$, $|B| = -6$ and $|C| = 4$. In each part, find the given determinant if possible.

**a)** $|B^5|$.
**b)** $|4A|$.
**c)** $|A + B|$.
**d)** $\left|(C^{-1})^T\right|$.
**e)** $\left|\frac{64}{5}A^{-1}\right|$.
**f)** $|AB^{-1}|$.

10) Let $A$ be a $3 \times 3$, $B$ be a $3 \times 4$ and $C$ be a $4 \times 3$ matrices with $|A| = 4$, $|BC| = 2$ and $|CB| = 3$. In each part, find the given determinant if possible.

**a)** $|B^4|$.
**b)** $|3ABC|$.
**c)** $|A + BC|$.
**d)** $\left|(C^T)^{-1}\right|$.
**e)** $|(ACB)^{-1}|$.
**f)** $\left|A(C^TB^T)^{-1}\right|$. (Recall: $(AB)^T = B^TA^T$.)

11) Let $A$ be a $2 \times 2$ and $B$ be a $2 \times 2$ matrices with $|A| = -2$ and $|B| = 3$. In each part, find the given determinant if possible.

**a)** $|B^4|$.
**b)** $|3A|$.
**c)** $|A + I_4|$.
**d)** $\left|(B^T)^{-1}\right|$.
**e)** $\left|\frac{2}{3}A^{-1}\right|$.
**f)** $|AB^{-1}|$.

12) Let $A$ and $B$ be $3 \times 4$ matrices with $\left|AB^T\right| = -2$ and $\left|A^TB\right| = 3$. In each part, find the given determinant if possible.

**a)** $|B^4|$.
**b)** $\left|(3A)B^T\right|$.
**c)** $|A + B|$.
**d)** $\left|A^{-1}(B^T)^{-1}\right|$.
**e)** $\left|\frac{2}{3}(BA^T)^{-1}\right|$.
**f)** $\left|(A^TB)^{-1}\right|$.