EXERCISE SET I

1) Write the volume of the solid region that lies between $z = x^2 + y^2$ and $z = 32 - x^2 - y^2$ as a triple integral in

(i) Cartesian Coordinates,

(ii) Cylindrical Coordinates,

(iii) Spherical Coordinates.

2) Write the volume of the solid region that is bounded from above by $x^2 + y^2 + z^2 = 16$ and from below by $z = \dfrac{1}{6}(x^2 + y^2)$ as a triple integral in

(i) Cartesian Coordinates,

(ii) Cylindrical Coordinates,

(iii) Spherical Coordinates.

3) Write the volume of the solid region that lies between $z = 16$ and $z = x^2 + y^2$ as a triple integral in

(i) Cartesian Coordinates,

(ii) Cylindrical Coordinates,

(iii) Spherical Coordinates.

4) Evaluate the line integral $\displaystyle\int_C (x^2 + y^2 - z^2)ds$ where C is the line segment from $(-1, 2, -3)$ to $(3, 5, 9)$.

5) Evaluate the line integral $\displaystyle\int_C \frac{2x^2 + y^2}{\sqrt{81y^2 + 16x^2}} ds$ where C is part of the ellipse $4x^2 + 9y^2 = 36$ in the first quadrant. (Hint: Consider $2x = 6\cos t$, $3y = 6\sin t$)

6) Evaluate the line integral $\displaystyle\int_C x^2dz + y^2dx - z^2dy$ where C is the line segment from $(-1, 2, -3)$ to $(3, 5, 9)$.

7) Evaluate the line integral $\displaystyle\int_C xdy - ydx$ where C is part of the ellipse $4x^2 + 9y^2 = 36$ in the first quadrant. (Hint: Consider $2x = 6\cos t$, $3y = 6\sin t$)

8) Let $\vec{F}(x, y, z) = \left(2x + y^2 + 3z - 3yz + 2xy^2z^2e^{x^2y^2z^2}\right)\vec{i}+$
$+\left(2xy - 3xz + 2x^2yz^2e^{x^2y^2z^2} + z^2 + \frac{2}{y}\right)\vec{j}+$
$+\left(3x - 3xy + \frac{1}{1+z^2} + 2x^2y^2ze^{x^2y^2z^2} + 2yz\right)\vec{k}.$

Evaluate the line integral $\displaystyle\int_C \vec{F} \bullet d\vec{r}$ where C is the curve with parametrization

$\vec{r}(t) = t^{155}\vec{i} + (1 + t^{156})\vec{j} + t^{255}\vec{k}$, with $0 \leq t \leq 1$.

9) Let $\vec{F}(x, y) = \left(2x\cos y - \dfrac{x}{\sqrt{x^2 + y^2}} + 2xe^{x^2}\right)\vec{i} + \left(2y - \dfrac{3}{y} - x^2 \sin y - \dfrac{y}{\sqrt{x^2 + y^2}}\right)\vec{j}$.

Evaluate the line integral $\displaystyle\int_C \vec{F} \bullet d\vec{r}$ where C is the portion of the curve $y = x^3 + 5x^2 - 4x + 2$ from $(0, 2)$ to $(1, 4)$.

10) Let D be the triangular region with vertices $(0, 0)$, $(1, 1)$, $(0, 2)$. Evaluate the line integral $\displaystyle\int_C (e^{x^2+2x} + x^2y)dx - (\cos(y^2) - xy + \ln(y^3 + 2))dy$ where C is the positively oriented boundary of the region D.

11) Let D be the region in the upper half plane bounded by the x-axis and the circle $x^2 + y^2 = 4$. Evaluate the line integral $\displaystyle\int_C (xy^2 - 99y + \sin(x^{2016}))dx + (x^2y + 156x - e^{y^2})dy$ where C is the positively oriented boundary of the region D.

12) Let D be the region bounded by the curves $y = x^2 - 2x + 5$ and $y = -x^2 + 6x + 5$. Evaluate the line integral $\displaystyle\int_C (3y - e^{x^3})dx - (x^2 + \tan(y^3 - 3y))dy$ where C is the positively oriented boundary of the region D.