SAMPLE QUESTIONS FOR THE FINAL EXAMINATION PART 2

1) Let $W = \{p(x) \in \mathbb{P}_4 \,|\, p(2) = 0,\ p(0) = 0\}$.

   (a) Show that W is a subspace of $\mathbb{P}_4$.

   (b) Find a basis S for W.

   (c) Find a basis B for $\mathbb{P}_4$ containing S.

2) For each $u = (u_1, u_2)$, $v = (v_1, v_2) \in \mathbb{R}^2$, define $(u,\ v) = u_1v_1 - 2u_1v_2 - 2u_2v_1 + 5u_2v_2$.

   a) Determine whether $(u,\ v)$ is an inner product or not.

   b) For $p(x) = (1, 2)$ and $v = (1, -1)$ evaluate (u, v).

3) For each $p(x),\ q(x) \in \mathbb{P}_2$, define $(p(x),\ q(x)) = \displaystyle\int_0^1 p(x)q(x)dx$.

   a) Determine whether $(p(x),\ q(x))$ is an inner product or not.

   b) For $p(x) = 2x + 1$ and $q(x) = 3x - 1$ evaluate $(p(x), q(x))$.

4) Let $V = \mathbb{R}^3$, $v_1 = (1, 2, 1)$, $v_2 = (1, 1, 1)$, $v_3 = (1, 1, -1)$, $w_1 = (1, 0, 0)$, $w_2 = (1, -1, 0)$, $w_3 = (1, 1, -1)$.

   a) Show that $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$.

   b) It is given that $T = \{w_1, w_2, w_3\}$ is a basis for $\mathbb{R}^3$. Find the transition matrix $P_{S \leftarrow T}$ from the basis T to the basis S.

   c) Compute the coordinate vector $[\alpha]_T$ of $\alpha = (3, 1, 2)$ with respect to T.

   d) Compute the coordinate vector $[\alpha]_S$ of $\alpha = (3, 1, 2)$ with respect to S.

5) Let $A = \begin{bmatrix} 1 & -2 & 2 & -1 & -1 & 2 \\ -1 & 2 & -1 & 0 & 0 & -1 \\ 2 & -4 & 3 & -1 & 0 & 5 \\ 3 & -6 & 4 & -1 & -3 & 4 \end{bmatrix}$ be a 4×6 matrix whose reduced row echelon form is $R = \begin{bmatrix} 1 & -2 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) Find a basis for the row space of A, if possible.

   (b) Find a basis for the column space of A, if possible.

   (c) Find a basis β for the solution space of the system $Ax = 0$.

   (d) Find the rank and nullity of A.

   (e) Find the vector α, where $[\alpha]_\beta = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$.

   (f) Determine whether or not $\gamma = (1, 2, -3, 1, 3, -1)$ is a solution of the system $Ax = 0$.

6) Let $v_1 = (-1, 1, 1)$, $v_2 = (1, 1, 2)$ and $v_3 = (-1, 1, -2)$. It is given that $T = \{v_1,\ v_2,\ v_3\}$ is a basis for $\mathbb{R}^3$.

(a) Find an orthonormal basis S for $\mathbb{R}^3$ by applying Gram-Schmidt orthogonalization process to the basis T.

(b) Find a basis for the orthogonal complement $W^{\perp}$ of the subspace $W = Span\{v_1,\ v_2\}$.