SAMPLE QUESTIONS FOR THE FINAL EXAMINATION PART 2

1) (a) Just show $W \neq \emptyset$ and it is closed under vector addition and scalar multiplication.

   (b) $S = \{\frac{-x^4}{2} + x^3,\ \frac{-x^4}{4} + x^2,\ \frac{-x^4}{8} + x,\ 1\}$.

   (c) $S = \{\frac{-x^4}{2} + x^3,\ \frac{-x^4}{4} + x^2,\ \frac{-x^4}{8} + x,\ 1,\ x\}$.

2) a) Show the following properties for all $u, v, w \in \mathbb{R}^2$ and $c \in \mathbb{R}$ using the definition.

   $(u, u) \geq 0$ and $(u, u) = 0$ if and only if $u = 0$

   $(u, v) = (v, u)$

   $(u, v + w) = (u, v) + (u, w)$

   $(cu, v) = c(u, v)$

   b) -11.

3) For each $p(x),\ q(x) \in \mathbb{P}_2$, define $(p(x),\ q(x)) = \int_0^1 p(x)q(x)dx$.

   a) Show the following properties for all $p(x), q(x), r(x) \in \mathbb{P}_2$ and $c \in \mathbb{R}$ using the definition.

   $(p(x), p(x)) \geq 0$ and $(p(x), p(x)) = 0$ if and only if $p(x) = 0$

   $(p(x), q(x)) = (q(x), p(x))$

   $(p(x), q(x) + r(x)) = (p(x), q(x)) + (p(x), r(x))$

   $(cp(x), q(x)) = c(p(x), q(x))$

   b) $\frac{3}{2}$.

4) a) Since $dim(\mathbb{R}^3) = 3$, it suffices to show that v_1, v_2, v_3 are linearly independent..

   b) $P_{S \leftarrow T} = \begin{bmatrix} -1 & -2 & 0 \\ 3/2 & 5/2 & 0 \\ 1/2 & 1/2 & 1 \end{bmatrix}$.

   c) $[\alpha]_T = \begin{bmatrix} 8 \\ -3 \\ -2 \end{bmatrix}$.

   d) $[\alpha]_S$ of $\alpha = (3, 1, 2) = \begin{bmatrix} -2 \\ 9/2 \\ 1/2 \end{bmatrix}$.

5) (a) $\{(1, -2, 0, 1, 0, -2), (0, 0, 1, -1, 0, 3), (0, 0, 0, 0, 1, 2)\}$

   (b) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -3 \end{bmatrix} \right\}$

   (c) $\beta = \{(2, 1, 0, 0, 0, 0), (-1, 0, 1, 1, 0, 0), (2, 0, -3, 0, -2, 1)\}$.

   (d) rank of A is 3 and nullity of A is 3.

   (e) $(8, 1, -10, 2, -8, 4)$

   (f) $\gamma = (1, 2, -3, 1, 3, -1)$ is not a solution of the system $Ax = 0$.

6) Let $v_1 = (-1, 1, 1)$, $v_2 = (1, 1, 2)$ and $v_3 = (-1, 1, -2)$. It is given that $T = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$.

(a) $\{(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (\frac{5}{\sqrt{42}}, \frac{1}{\sqrt{42}}, \frac{4}{\sqrt{42}}), (\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-2}{\sqrt{14}})\}$.

(b) $\{(1, 3, -2)\}$.