

Exercise Set 1 ANSWERS

1) i) $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{x^2+y^2}^{32-x^2-y^2} dz dy dx$ OR $\int_{-4}^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} \int_{x^2+y^2}^{32-x^2-y^2} dz dx dy$

OR $\int_{-4}^4 \int_{x^2}^{16} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dy dz dx + \int_{-4}^4 \int_{16}^{32-x^2} \int_{-\sqrt{32-x^2-z}}^{\sqrt{32-x^2-z}} dy dz dx$

OR $\int_{-4}^4 \int_{y^2}^{16} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx dz dy + \int_{-4}^4 \int_{16}^{32-y^2} \int_{-\sqrt{32-y^2-z}}^{\sqrt{32-y^2-z}} dx dz dy$

OR $\int_0^{16} \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dy dx dz + \int_{16}^{32} \int_{-\sqrt{32-z}}^{\sqrt{32-z}} \int_{-\sqrt{32-x^2-z}}^{\sqrt{32-x^2-z}} dy dx dz$

OR $\int_0^{16} \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} dx dy dz + \int_{16}^{32} \int_{-\sqrt{32-z}}^{\sqrt{32-z}} \int_{-\sqrt{32-y^2-z}}^{\sqrt{32-y^2-z}} dx dy dz$

ii) $\int_0^{2\pi} \int_0^4 \int_{r^2}^{32-r^2} r dz dr d\theta$ OR $\int_0^4 \int_0^{2\pi} \int_{r^2}^{32-r^2} r dz d\theta dr$

OR $\int_0^{2\pi} \int_0^{16} \int_0^{\sqrt{z}} r dr dz d\theta + \int_0^{2\pi} \int_{16}^{32} \int_0^{\sqrt{32-z}} r dr dz d\theta$

OR $\int_0^4 \int_{r^2}^{32-r^2} \int_0^{2\pi} r d\theta dz dr$

OR $\int_0^{16} \int_0^{2\pi} \int_0^{\sqrt{z}} r dr d\theta dz + \int_{16}^{32} \int_0^{2\pi} \int_0^{\sqrt{32-z}} r dr d\theta dz$

OR $\int_0^{16} \int_0^{\sqrt{z}} \int_0^{2\pi} r d\theta dr dz + \int_{16}^{32} \int_0^{\sqrt{32-z}} \int_0^{2\pi} r d\theta dr dz$

$$\text{iii)} \int_0^{2\pi} \int_0^{\arctan \frac{1}{4}} \int_0^{\frac{-\cos \phi + \sqrt{\cos^2 \phi + 128 \sin^2 \phi}}{2 \sin^2 \phi}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi} \int_{\arctan \frac{1}{4}}^{\frac{\pi}{2}} \int_0^{\frac{\cos \phi}{\sin^2 \phi}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

$$2) \text{ i)} \int_{-2\sqrt{3}}^{2\sqrt{3}} \int_{-\sqrt{12-x^2}}^{\sqrt{12-x^2}} \int_{\frac{1}{6}(x^2+y^2)}^{\sqrt{16-x^2-y^2}} dz \, dy \, dx$$

$$\text{ii)} \int_0^{2\pi} \int_0^{2\sqrt{3}} \int_{\frac{r^2}{6}}^{\sqrt{16-r^2}} r \, dz \, dr \, d\theta$$

$$\text{iii)} \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^4 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{\frac{6 \cos \phi}{\sin^2 \phi}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

$$3) \text{ i)} \int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{x^2+y^2}^{16} dz \, dy \, dx$$

$$\text{ii)} \int_0^{2\pi} \int_0^4 \int_{r^2}^{16} r \, dz \, dr \, d\theta$$

$$\text{iii)} \int_0^{2\pi} \int_0^{\arctan \frac{1}{4}} \int_0^{\frac{16}{\cos \phi}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi} \int_{\arctan \frac{1}{4}}^{\frac{\pi}{2}} \int_0^{\frac{\cos \phi}{\sin^2 \phi}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

$$4) -\frac{221}{3}$$

$$7) 3\pi$$

$$5) \frac{11\pi}{12}$$

$$8) 3 + e^4 + \ln 4 + \frac{\pi}{4}$$

$$6) 17$$

$$9) \cos 4 - \sqrt{17} + e + 13 - \ln 8$$

$$10) \frac{5}{6}$$

$$11) 510\pi$$

$$12) -\frac{43}{3}$$