Answers (not solutions) of Exercise Set 1

1) a) $\begin{bmatrix} 0 & 1 & -1 & 1 & 5 & 5 \\ 3 & 3 & 3 & 1 & 0 & 6 \\ 1 & 2 & 0 & 1 & 4 & 6 \\ 2 & -1 & 5 & 0 & -8 & -4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 2 & 0 & -3 & -1 \\ 0 & 1 & -1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\{ \begin{bmatrix} -1-2z+3w \\ 2+z-2w \\ z \\ 3-3w \\ w \end{bmatrix} | z, w \in \mathbb{R} \}$

2) a) $\begin{bmatrix} 2 & -1 & 0 & 5 & 4 \\ 1 & -1 & 1 & 6 & 0 \\ 3 & -2 & 2 & 14 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 3 & -1 \end{bmatrix}$

c) $\{ \begin{bmatrix} 3-2t \\ 2+t \\ -1-3t \\ t \end{bmatrix} | t \in \mathbb{R} \}$

3) a) $\begin{bmatrix} 2 & -2 & -1 & 1 & 2 & 1 & -6 \\ -3 & 3 & -1 & 0 & -7 & 2 & -3 \\ 1 & -1 & 1 & 0 & 3 & -1 & 3 \\ 1 & 0 & 1 & 0 & 1 & -1 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -\frac{1}{2} & 3 \\ 0 & 0 & 0 & 1 & -1 & \frac{3}{2} & -3 \end{bmatrix}$

c) $\{ \begin{bmatrix} -1+\frac{1}{2}w \\ 1+2t \\ 3-t+\frac{1}{2}w \\ -3+t-\frac{3}{2}w \\ t \\ w \end{bmatrix} | t, w \in \mathbb{R} \}$

4) a) $\begin{bmatrix} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 8 \end{bmatrix}$

c) $\{ \begin{bmatrix} -2+t \\ -1 \\ 8-2t \\ t \end{bmatrix} | t \in \mathbb{R} \}$

5) a) $A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{17}{15} & -\frac{1}{3} \\ -\frac{1}{5} & -\frac{14}{15} & \frac{1}{3} \\ -\frac{2}{5} & -\frac{8}{15} & \frac{1}{3} \end{bmatrix}$

b) $x = \begin{bmatrix} \frac{69}{15} \\ \frac{-63}{15} \\ \frac{-36}{15} \end{bmatrix}$

6) a) $A^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ -\frac{2}{11} & -\frac{1}{11} & \frac{9}{11} \\ \frac{4}{11} & \frac{2}{11} & -\frac{7}{11} \end{bmatrix}$

b) $x = \begin{bmatrix} 11 \\ 8 \\ -5 \end{bmatrix}$

7) a) $A^{-1} = \begin{bmatrix} -\frac{2}{5} & 1 & -\frac{1}{5} \\ -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{11}{15} & -\frac{2}{3} & \frac{1}{5} \end{bmatrix}$

b) $x = \begin{bmatrix} 4 \\ \frac{-7}{3} \\ \frac{-4}{3} \end{bmatrix}$

8) a) $A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{2} & \frac{2}{5} \\ -\frac{8}{5} & -\frac{5}{2} & -\frac{3}{5} \\ \frac{1}{5} & \frac{1}{2} & \frac{1}{5} \end{bmatrix}$

b) $x = \begin{bmatrix} \frac{-7}{10} \\ \frac{3}{10} \\ \frac{-1}{10} \end{bmatrix}$

9) $3a - b + c = 0$

10) a) If $a = -2$ then the system has no solution.

b) If $a \neq 2$ and $a \neq -2$ then the system has a unique solution.

c) If $a = -2$ then the system has infinitely many solutions.

11) If $a = 6$ then the matrix is not invertible (i.e. it is singular).