MATH 255 - Vector Calculus and Linear Algebra

EXERCISE SET 1

1. As a triple integral in Cylindrical Coordinates, write the volume of the solid region that lies between $z = x^2 + y^2$ and $z = 32 - x^2 - y^2$. Find the volume.

2. As a triple integral in Spherical Coordinates, write the volume of the solid region that is bounded from above by $x^2+y^2+z^2 = 16$ and from below by $z^2 = \dfrac{1}{3}(x^2+y^2)$. Find the volume.

3. As a triple integral in Cylindrical Coordinates, write the volume of the solid region that lies between $z = 16$ and $z = x^2 + y^2$. Find the volume.

4. As a triple integral in Spherical Coordinates, write the volume of part of the sphere $x^2+y^2+z^2 = 16$ lying below the plane $z = 2$. Find the volume.

5. Let D be the solid region that is bounded from above by $x^2 + y^2 + z^2 = 16$ and from below by $z = 2\sqrt{3}$. Write the integral $\iiint_D f(x, y, z)dV$ as an iterated triple integral in Spherical Coordinates.

6. Let D be the solid region that lies between $z = -\sqrt{3x^2 + 3y^2}$ and $z = 6 - x^2 - y^2$. Write the integral $\iiint_D f(x, y, z)dV$ as an iterated triple integral in Cylindrical Coordinates.

7. Let D be the solid region that is bounded from below by $x^2 + y^2 + z^2 = 16$ and from above by $z^2 = \dfrac{1}{3}(x^2+y^2)$. Write the integral $\iiint_D f(x, y, z)dV$ as an iterated triple integral in Spherical Coordinates.

8. Evaluate the line integral $\displaystyle\int_C (x^2 + y^2 - z^2)ds$ where C is the line segment from $(-1, 2, -3)$ to $(3, 5, 9)$.

9. Evaluate the line integral $\displaystyle\int_C \frac{2x^2 + y^2}{\sqrt{81y^2 + 16x^2}}ds$ where C is part of the ellipse $4x^2 + 9y^2 = 36$ in the first quadrant. (Hint: Consider $2x = 6\cos t$, $3y = 6\sin t$)

10. Evaluate the line integral $\displaystyle\int_C x^2dz + y^2dx - z^2dy$ where C is the line segment from $(-1, 2, -3)$ to $(3, 5, 9)$.

11. Evaluate the line integral $\displaystyle\int_C xdy - ydx$ where C is part of the ellipse $4x^2 + 9y^2 = 36$ in the first quadrant. (Hint: Consider $2x = 6\cos t$, $3y = 6\sin t$)

12. Let $\vec{F}(x, y, z) = \left(2x + y^2 + 3z - 3yz + 2xy^2z^2e^{x^2y^2z^2}\right)\vec{i}+$
$+\left(2xy - 3xz + 2x^2yz^2e^{x^2y^2z^2} + z^2 + \frac{2}{y}\right)\vec{j}+$
$+\left(3x - 3xy + \frac{1}{1+z^2} + 2x^2y^2ze^{x^2y^2z^2} + 2yz\right)\vec{k}.$

Evaluate the line integral $\int\limits_C \vec{F} \bullet d\vec{r}$ where C is the curve with parametrization $\vec{r}(t) = t^{155}\vec{i} + (1+t^{156})\vec{j} + t^{255}\vec{k}$, with $0 \leq t \leq 1$.

13. Let $\vec{F}(x,y) = \left(2x\cos y - \dfrac{x}{\sqrt{x^2+y^2}} + 2xe^{x^2}\right)\vec{i} + \left(2y - \dfrac{3}{y} - x^2\sin y - \dfrac{y}{\sqrt{x^2+y^2}}\right)\vec{j}$.

    Evaluate the line integral $\int\limits_C \vec{F} \bullet d\vec{r}$ where C is the portion of the curve $y = x^3 + 5x^2 - 4x + 2$ from $(0,2)$ to $(1,4)$.

14. Let D be the triangular region with vertices $(0,0)$, $(1,1)$, $(0,2)$. Evaluate the line integral $\int\limits_C (e^{x^2+2x} + x^2y)dx - (\cos(y^2) - xy + \ln(y^3+2))dy$ where C is the positively oriented boundary of the region D.

15. Let D be the region in the upper half plane bounded by the x-axis and the circle $x^2+y^2=4$. Evaluate the line integral $\int\limits_C (xy^2 - 99y + \sin(x^{2016}))dx + (x^2y + 156x - e^{y^2})dy$ where C is the positively oriented boundary of the region D.

16. Let D be the region bounded by the curves $y = x^2 - 2x + 5$ and $y = -x^2 + 6x + 5$. Evaluate the line integral $\int\limits_C (3y - e^{x^3})dx - (x^2 + \tan(y^3 - 3y))dy$ where C is the positively oriented boundary of the region D.

ANSWERS (Not the Solutions)

1. $\displaystyle\int_0^{2\pi}\int_0^4\int_{r^2}^{32-r^2} r\,dz\,dr\,d\theta = 256\pi$

2. $\displaystyle\int_0^{2\pi}\int_0^{\pi/3}\int_0^4 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{64\pi}{3}$

3. $\displaystyle\int_0^{2\pi}\int_0^4\int_{r^2}^{16} r\,dz\,dr\,d\theta = 128\pi$

4. $\displaystyle\int_0^{2\pi}\int_0^{\pi/3}\int_0^{\frac{2}{\cos\phi}} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta + \int_0^{2\pi}\int_{\pi/3}^{\pi}\int_0^4 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = 72\pi$

5. $\displaystyle\int_0^{2\pi}\int_0^{\pi/6}\int_{\frac{2\sqrt{3}}{\cos\phi}}^4 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

6. $\displaystyle\int_0^{2\pi}\int_0^{2}\sqrt{3}\int_{-\sqrt{3}r}^{6-r^2} r\,dz\,dr\,d\theta$

7. $\displaystyle\int_0^{2\pi}\int_{\pi/3}^{\pi}\int_0^4 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

8. $\frac{-221}{3}$

9. $\frac{11\pi}{12}$

10. 17
11. 3π
12. $3 + e^4 + \ln 4 + \frac{\pi}{4}$
13. $\cos 4 - \sqrt{17} + e + 13 - \ln 8$
14. $\frac{5}{6}$
15. 510π
16. $\frac{-448}{3}$