MATH 255 - Vector Calculus and Linear Algebra

EXERCISE SET 4

1) Let S be the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upward. Let $\vec{F} = y^2\vec{i} + x\vec{j} + z^2\vec{k}$.

   a) Evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$ directly.

   b) Use Stoke's theorem to evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$.

   ANSWER: a) π b) π

2) Let S be the part of the cone $z = \sqrt{x^2 + y^2}$ bounded by the plane $z = 4$, oriented downward. Let $\vec{F} = -y\vec{i} + x\vec{j} - 2\vec{k}$.

   a) Evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$, directly.

   b) Use Stoke's theorem to evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$.

   ANSWER: a) -32π b) -32π

3) Let S be the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane $z = 1$, oriented upward. Let $\vec{F} = -2yz\vec{i} + y\vec{j} + 3x\vec{k}$.

   a) Evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$, directly.

   b) Use Stoke's theorem to evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$.

   ANSWER: a) 8π b) 8π

4) Let S be the hemisphere $x^2 + y^2 + z^2 = 1, \quad y \geq 0$, oriented in the direction of positive y-axis. Let $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$.

   a) Evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$, directly.

   b) Use Stoke's theorem to evaluate $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \bullet \vec{n}dS$.

   ANSWER: a) $-\pi$ b) $-\pi$