MATH 255 - Vector Calculus and Linear Algebra

EXERCISE SET 5

1. Consider the system of linear equations

$$\begin{array}{rcrcrcrcrcr} & + & y & - & z & + & t & + & 5w & = & 5 \\ 3x & + & 3y & + & 3z & + & t & & & = & 6 \\ x & + & 2y & & & + & t & + & 4w & = & 6 \\ 2x & - & y & + & 5z & & & - & 8w & = & -4 \end{array}$$

   (a) Write the augmented matrix of the system.

   (b) Find the reduced row echelon form of the augmented matrix you found in part (a).

   (c) Find the set of solutions of the system.

2. Consider the system of linear equations

$$\begin{array}{rcrcrcrcr} 2x & - & y & & & + & 5t & = & 4 \\ x & - & y & + & z & + & 6t & = & 0 \\ 3x & - & 2y & + & 2z & + & 14t & = & 3 \end{array}$$

   (a) Write the augmented matrix of the system.

   (b) Find the reduced row echelon form of the augmented matrix you found in part (a).

   (c) Find the set of solutions of the system.

3. Consider the system of linear equations

$$\begin{array}{rcrcrcrcrcrcr} 2x & - & 2y & - & z & + & s & + & 2t & + & w & = & -6 \\ -3x & + & 3y & - & z & & & - & 7t & + & 2w & = & -3 \\ x & - & y & + & z & & & + & 3t & - & w & = & 3 \\ x & & & + & z & & & + & t & - & w & = & 4 \end{array}$$

   (a) Write the augmented matrix of the system.

   (b) Find the reduced row echelon form of the augmented matrix you found in part (a).

   (c) Find the set of solutions of the system.

4. Consider the system of linear equations

$$\begin{array}{rcrcrcrcr} x & + & y & + & 2z & + & 3t & = & 13 \\ x & - & 2y & + & z & + & t & = & 8 \\ 3x & + & y & + & z & - & t & = & 1 \end{array}$$

   (a) Write the augmented matrix of the system.

   (b) Find the reduced row echelon form of the augmented matrix you found in part (a).

   (c) Find the set of solutions of the system.

5. Let $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using elementary row operations.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

6. Let $A = \begin{bmatrix} -1 & 4 & 2 \\ 2 & -1 & 5 \\ 0 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 11 \\ -11 \\ 11 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using elementary row operations.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

7. Let Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 7 & 8 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 3 \\ -15 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using elementary row operations.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

8. Let $A = \begin{bmatrix} 2 & -1 & -7 \\ -2 & 0 & 4 \\ 3 & 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using elementary row operations.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

9. Find an equation relating a, b, and c so that the linear system

$$\begin{array}{rcrcrcl} 2x & + & 2y & + & 3z & = & a \\ 5x & + & y & + & 8z & = & b \\ -x & - & 5y & - & z & = & c \end{array}$$

   is consistent for any values of a, b and c that satisfy that equation.

10. Find all values of a, for which the following linear system has

    (a) no solution;

    (b) a unique solution;

    (c) infinitely many solutions;

$$\begin{array}{rcrcrcl} x & + & y & - & z & = & 2 \\ 2x & + & 3y & & & = & 5 \\ -x & - & 2y & + & (a^2-5)z & = & a-5 \end{array}$$

11. Let $A = \begin{bmatrix} 3 & -1 & 5 \\ 1 & -2 & 1 \\ 2 & 6 & a \end{bmatrix}$. Find the value(s) of a so that A is not invertible.