MATH 255 - Vector Calculus and Linear Algebra

EXERCISE SET 6

1. Let $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using adjoint of A.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

   (c) Find the solution of $Ax = b$ by using Cramer's Rule if A is invertible.

2. Let $A = \begin{bmatrix} -1 & 4 & 2 \\ 2 & -1 & 5 \\ 0 & 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 11 \\ -11 \\ 11 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using adjoint of A.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

   (c) Find the solution of $Ax = b$ by using Cramer's Rule if A is invertible.

3. Let Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 7 & 8 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 3 \\ -15 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using adjoint of A.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

   (c) Find the solution of $Ax = b$ by using Cramer's Rule if A is invertible.

4. Let $A = \begin{bmatrix} 2 & -1 & -7 \\ -2 & 0 & 4 \\ 3 & 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

   (a) Find A^{-1} if A is invertible, by using adjoint of A.

   (b) Find the solution of $Ax = b$ by using A^{-1} if A is invertible.

   (c) Find the solution of $Ax = b$ by using Cramer's Rule if A is invertible.

5. Let $W = \{p(x) \in \mathbb{P}_4 \,|\, p(2) = 0,\ p(0) = 0\}$.

   (a) Show that W is a subspace of $\mathbb{P}_4$.

   (b) Find a basis S for W.

   (c) Find a basis B for $\mathbb{P}_4$ containing S.

6. For each $u = (u_1, u_2)$, $v = (v_1, v_2) \in \mathbb{R}^2$, define $(u,\ v) = u_1v_1 - 2u_1v_2 - 2u_2v_1 + 5u_2v_2$.

   a) Determine whether $(u,\ v)$ is an inner product or not.

   b) For $p(x) = (1, 2)$ and $v = (1, -1)$ evaluate (u, v).

7. For each $p(x)$, $q(x) \in \mathbb{P}_2$, define $(p(x),\, q(x)) = \displaystyle\int_0^2 p(x)q(x)dx$.

a) Determine whether $(p(x),\, q(x))$ is an inner product or not.

b) For $p(x) = 2x + 1$ and $q(x) = 3x - 1$ evaluate $(p(x), q(x))$.

8. Let $V = \mathbb{R}^3$, $v_1 = (1, 2, 1)$, $v_2 = (1, 1, 1)$, $v_3 = (1, 1, -1)$. Show that $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$.

9. Let $A = \begin{bmatrix} 1 & -2 & 2 & -1 & -1 & 2 \\ -1 & 2 & -1 & 0 & 0 & -1 \\ 2 & -4 & 3 & -1 & 0 & 5 \\ 3 & -6 & 4 & -1 & -3 & 4 \end{bmatrix}$ be a 4×6 matrix whose reduced row echelon form is

$$R = \begin{bmatrix} 1 & -2 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the row space of A, if possible.

(b) Find a basis for the column space of A, if possible.

(c) Find a basis β for the solution space of the system $Ax = 0$.

(d) Find the rank and nullity of A.

(e) Determine whether or not $\gamma = (1, 2, -3, 1, 3, -1)$ is a solution of the system $Ax = 0$.