

Math 255 Vector Calculus and
Linear Algebra

Exercise Set 3

1) Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 4$.
(ANSWER: $16\pi\sqrt{2}$)

2) Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$.
(ANSWER: $\frac{17\sqrt{17}-1}{6}\pi$)

3) Find the surface area of the portion of the plane $3xy + 2z = 6$ inside the cylinder $x^2 + y^2 = 4$.
(ANSWER: $2\pi\sqrt{14}$)

4) Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ above the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$.
(ANSWER: $\frac{\sqrt{2}}{2}$)

5) Find the surface area of the portion of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ above the plane $z = 1$.
(ANSWER: 4π)

6) Evaluate $\iint_S z \, dS$ where S is the portion of the plane $z = 2x + 3y$ above the rectangle $1 \leq x \leq 2$, $1 \leq y \leq 3$
(ANSWER: $\frac{82\sqrt{14}}{3}$)

7) Evaluate $\iint_S (x^2 + y^2 + z^2)^{3/2} \, dS$ where S is the lower hemisphere $z = -\sqrt{9 - x^2 - y^2}$.
(ANSWER: 486π)

8) Evaluate $\iint_S (x^2 + y^2 - z) \, dS$ where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ between $z = 1$ and $z = 2$.
(ANSWER: $\frac{\pi}{10}(81 - 13\sqrt{13})$)

9) Use Divergence Theorem to compute $\iint_S \vec{F} \cdot \vec{n} dS$ where

S is the boundary of Q , the solid region bounded by $x+y+2z=2$ (in the first octant) and the coordinate planes, \vec{n} is the outward unit normal vector and $\vec{F} = \langle 2x-y^2, 4xz-2y, xy^3 \rangle$.

(ANSWER: 0)

10) Use Divergence Theorem to compute $\iint_S \vec{F} \cdot \vec{n} dS$ where

S is the boundary of Q , the solid region bounded by the planes $x=0, x=2, y=1, y=2, z=-1, z=2$, \vec{n} is the outward unit normal vector and $\vec{F} = \langle y^3-2x, e^{x^2}, 4z \rangle$.

(ANSWER: 12)

11) Use Divergence Theorem to compute $\iint_S \vec{F} \cdot \vec{n} dS$ where

S is the boundary of Q , the solid region bounded by $z=x^2+y^2$ and $z=4$, \vec{n} is the outward unit normal vector, and $\vec{F} = \langle x^3, y^3-z, xy^2 \rangle$

(ANSWER: 32π)

12) Use Divergence Theorem to compute $\iint_S \vec{F} \cdot \vec{n} dS$ where

S is the boundary of Q , the solid region bounded by $z=\sqrt{x^2+y^2}$ and $z=\sqrt{2-x^2-y^2}$, \vec{n} is the outward unit normal vector and $\vec{F} = \langle x^2, z^2-x, y^3 \rangle$.

(ANSWER: 0)

13) Use Divergence theorem to compute $\iint_S \vec{F} \cdot \vec{n} dS$ where

S is the boundary of Q , the solid region bounded by $z=\sqrt{x^2+y^2}$ and $x^2+y^2=1$ and $z=0$, \vec{n} is the outward unit normal vector, and $\vec{F} = \langle y^2, x^2z, z^2 \rangle$.

(ANSWER: $\frac{\pi}{2}$).